Introduction to Neural Codes, Rings, and Ideals

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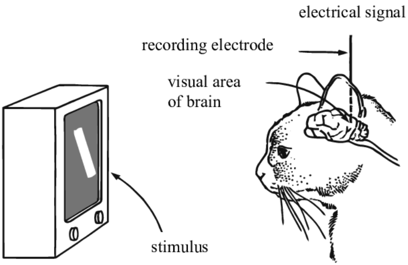
Table of Contents

1959 - Hubel & Wiesel

- By shining small spots of light on the light-adapted cat retina showed that *ganglion cells* have concentric *receptive fields*, with an 'on' center and an 'off' periphery, or vice versa.



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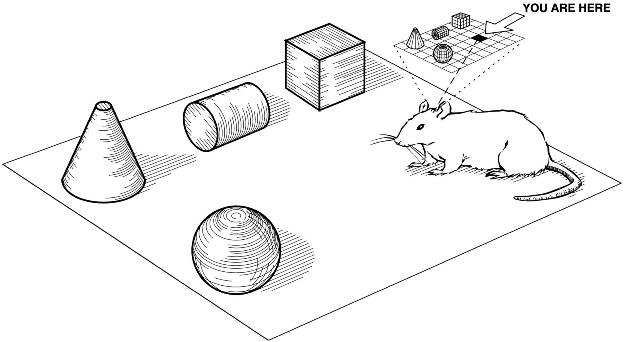
- the activity of these neurons changed depending on the orientation of the line
- sometimes the neurons fired only when the line was moving in a particular direction.

1971- O'Keefe & Dostorvsky

- "We do not experience the world as a stream of unrelated stimuli; rather, our brains organize different types of stimuli into highly structured stimulus spaces" (Y.2013)



1971- O'Keefe & Dostorvsky

- discovered a specific class of neurons in the hippocampus called *place cells*, as they were very active in response to changes in the spatial surroundings such as changes in color or shape or the introduction or removal of items.

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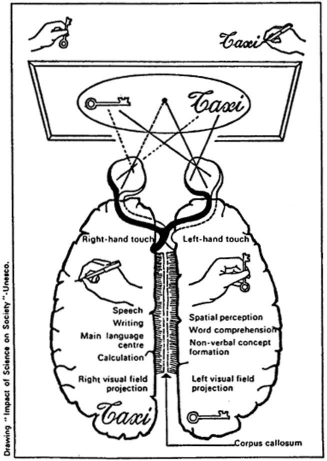
- the place cells would fire when the rat entered a specific area, the place field of the neuron.
- different place cells correspond to different place fields, and these place fields may overlap or cover other place fields.

1989- Sperry


Taxi
Taxi
Right-hand touch
Left-hand touch
Speech
Writing
Main language centre
Calculation
Right visual field projection
Taxi
Spatial perception
Word comprehension
Non-verbal concept formation
Left visual field projection
Corpus callosum
Drawing "Impact of Science on Society"-Unesco.

The Neural Code

- Given a set of neurons labelled $\{1, .., n\} = [n]$, we define a *neural code* $\mathcal{C} \subset \{0, 1\}$ as a set of binary patterns of neural activity.
- An element of a neural code is called a *codeword* $c = (c_1, \ldots, c_n) \in \mathcal{C}$ and corresponds to a subset of neurons $\text{supp}(c) = \{i \in [n] \mid c_i = 1\} \subset [n]$.

Example of Neural Code

$$\mathcal{C} = \{000, 100, 010, 110, 001\}$$

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$$\text{supp } \mathcal{C} = \{\emptyset, \{1\}, \{2\}, \{1,2\}, \{3\}\}$$

Receptive Fields

For a *stimulus space* X, we define the **receptive field** to be the subset U_i of X in which neuron i fires.

Example of Receptive Field Codes

- **Orientation-selective** neurons have *tuning curves* that reflect a neuron's preference for a particular angle.

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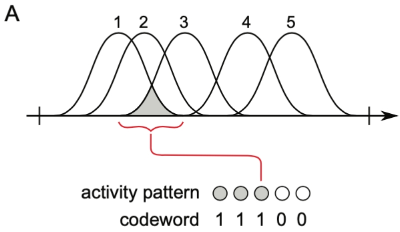

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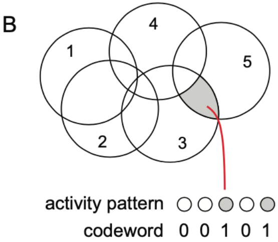

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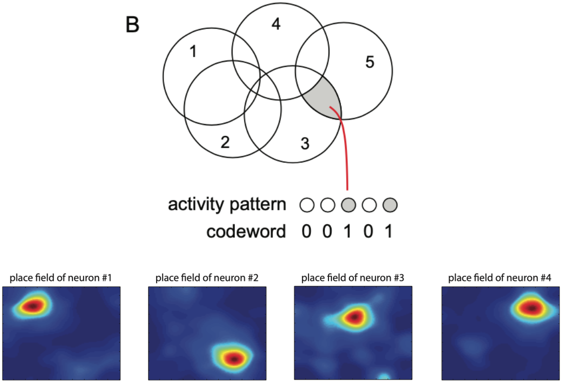

Basic Definitions

- A **ring** R is a set which is closed under "multiplication" and "addition" and has an identity element such that:
  - the distributive property holds
  - multiplication is associative
  - there is an identity element for multiplication and addition

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  From now on $k = \mathbb{F}_2$, $R = k[x_1, \ldots, x_n]$.

- Let J be an ideal of R and define the **corresponding variety**

$$V(J) = \{v \in k^n \mid f(v) = 0, \forall f \in J\}.$$

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$$V(J) = \{v \in k^n \mid f(v) = 0, \forall f \in J\}.$$

- Let S be a subset of k^n and define

$$I(S) = \{f \in R \mid f(v) = 0 \forall, v \in S\}$$

The Neural Ideal

- Let $\mathcal{C}$ be a neural code, and define the **neural ideal** (naive approach) $I_{\mathcal{C}}$ as

$$I(\mathcal{C}) = \{f \in R \mid f(c) = 0, \forall c \in \mathcal{C}\}$$

The Neural Ring

- The **neural ring** $R_{\mathcal{C}}$ corresponding to the code $\mathcal{C}$ is the quotient ring

$$R_{\mathcal{C}} = R/I_{\mathcal{C}}$$

Connections between algebra, combinatorics, and biology

- Polarization of neural code

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- Partial Code

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- Polarization of neural code
- Partial Code
- Mirror Neurons

References

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